Recitation 7

Big O's and Pigeons

Review

Defn 1: (Big O). We say $f(x) \in O(g(x))$ if g grows 'faster' than f.

Formally, if $f(x) \in O(g(x))$ then $\exists c, k$ such that $|f(x)| < c|g(x)|$ for all $x > k$.

Thm: (Pigeonhole principle). If we take k + 1 pigeons, and put them into k holes, some hole must contain at least two pigeons.

More generally, if we put n objects into k boxes, then some box has at least $\lceil n/k \rceil$ objects.

Warm-Up

a. Answer true or false for all of the following

i. The relation $R = \{(f, g) \mid f(n) \in O(g(n))\}$ is reflexive.
ii. The relation $R = \{(f, g) \mid f(n) \in O(g(n))\}$ is transitive.
iii. The relation $R = \{(f, g) \mid f(n) \in O(g(n))\}$ is an equivelance relation.
iv. $2n \in O(n)$
v. $2n \in O(n^2)$
vi. $n^3 \in O(n^2)$
vii. $100n^2 \in O(n^2)$
viii. $n^{100} \in O(2^n)$

b. $f : A \to B$ where $|A| > |B|$ and A and B are finite. Show f is not an injection.

c. $f : A \to B$ where $|A| = |B|$ where A and B are finite. Show that if f is not a surjection then f is not an injection.

Checkpoint - Call over a TA

d. There are 9 planes and 13 airports. Each day every plane visits 3 different airports. Prove that there must exist one airport each day which is visited by at least 3 planes.

e. Given an arbitrary sequence of 100 integers, prove that there exists a consecutive subsequence whose sum is divisible by 100.

Hint: Start by considering consecutive subsequences starting at the first element.

Checkpoint - Call over a TA

Pigeonhole Problems

Ice Cream Social Problem

There are n people at the ice cream social. Throughout the night they have a series of dance partners.

i. The minimum number of dance partners someone can have is 0. What is the maximum number of dance partners?

ii. Prove that 2 people will have the same number of dance partners by the end of the night.